

Design of an Observer and Speed Controller for a DC Motor Fed by Fuel Cells and DC to DC Converters

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Abstract – This paper presents a method to control the speed of a separately excited D.C. motor using a D.C. to D.C. converter. The energy source of the system is a Fuel Cell (FC), for which a mathematical model describing its electrical characteristics is also presented. By controlling the duty cycle of the converter, the input power of the motor is also controlled; thus controlling the speed of the motor. Depending on the operating characteristics of the FC used, the motor may or may not be controlled due to the restrictions imposed on the duty cycle. The range of existence of the duty cycle is derived based on the analysis of impedances of the system. A state observer is designed for sensorless current estimation. Simulations are made using the system's dynamic equations to demonstrate the obtained results.

Index Terms - buck boost converter, duty cycle, fuel cells, motor PWM, renewable energy, speed control, observer

I. INTRODUCTION

A fuel cell (FC) is an electrochemical device that dynamically converts the energy of a chemical reaction between a fuel, commonly hydrogen, and an oxidant into electrical energy for our consumption [1]. In principle, a fuel cell operates like a battery consisting of an electrolyte placed between two electrodes: an anode and a cathode. They differ from batteries in that they are designed for continuous replenishment of the reactants consumed (i.e. they do not run down or require charging). A more detailed description of how a fuel cell works and its history can be found in [16]. FC's are one of the most promising sources of renewable energy, especially in the automotive industry now that the scientific community is looking for new and better renewable energy sources to substitute fossil fuels. Because of environmental and economic problems, the interest to develop more electric/hybrid cars has increased. This particular application is closely related to the field of power electronics since the electric power supplied by any source must be controlled and conditioned before being applied to the motor of an electric car. The system should be able to operate at controlled power levels in order for the acceleration of the load to be controlled, thus controlling the speed of the car. The ability to precisely control this speed is crucial in order to achieve a safe and practical electric vehicle design. This paper focuses on a system that uses a fuel cell (FC) as its energy source and a Buck-Boost converter as the power signal conditioner that

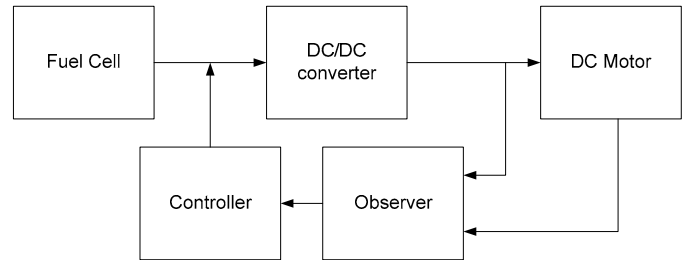


Fig. 1. System block diagram.

drives the load, in this case a D.C. motor.

It is desired to control the speed of a separately excited D.C. motor connected to an FC through a voltage converter. The control system will relate the angular velocity of the motor's shaft with the converter's duty cycle. Maximum power point tracking (MPPT) in a similar system energized by photovoltaic (PV) panels has been a topic of interest in past research [6]–[8]. In the cases presented in [3], [9]; MPPT is used to extract as much power as possible from the FC. In reference [12], a control law relating the duty cycle and the charging current has been developed. A similar method is used to derive a control system to regulate the load's velocity by controlling the converter's duty cycle.

II. PROPOSED SYSTEM

A. Fuel Cell Mathematical Model for Electric Characterization

The proposed electrical FC mathematical model considers the boundary conditions and shape of the FC's P-V curve (Fig. 2). The mathematical model used is presented in (1). The variables V_h , I_h and P_{max} are the maximum voltage, maximum current and maximum power for the FC, respectively. The variable k is the characteristic constant for the fuel cell based on the V-I and P-I curves as given by (2). P_{max} is obtained by multiplying the optimal voltage times the optimal current (V_{op} and I_{op}).

$$V = V_L + (V_H - V_L) \cdot \left[\frac{\cos^{-1} \left(\frac{2 \cdot I}{I_H} - 1 \right)}{\pi} \right]^k \quad (1)$$

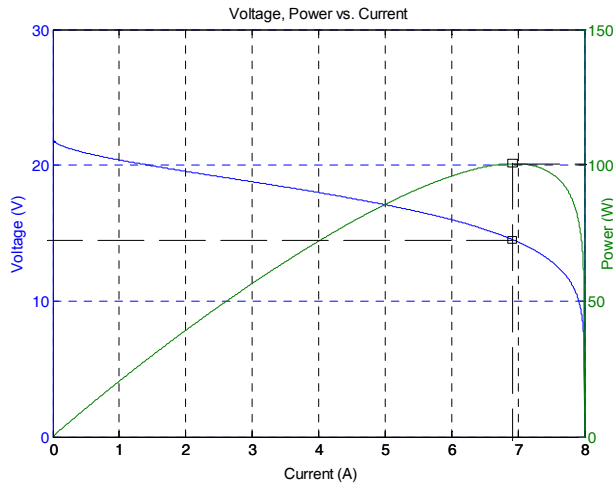


Fig.2. Voltage vs. current and power vs. current curves for the fuel cell using the proposed model.

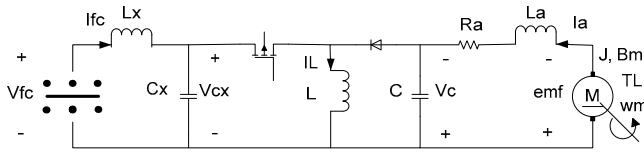


Fig.3. Detailed system diagram, excluding controller and observers

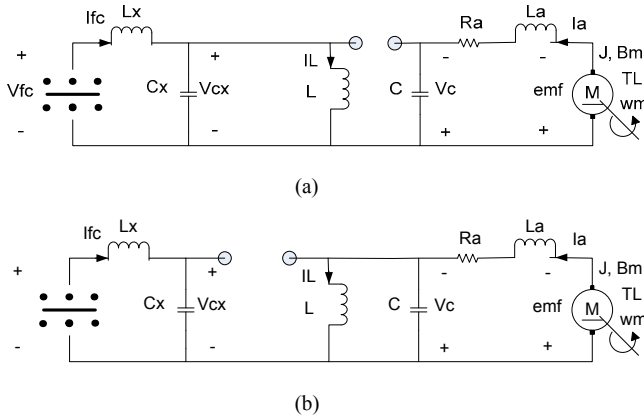


Fig.4. The two states of the system: (a) when the MOSFET is on; and (b) when it is off.

$$k = \ln \left[\frac{P_{max} - I_{op} \cdot V_L}{I_{op} \cdot V_H - I_{op} \cdot V_L} \right] / \ln \left[\frac{\arccos \left(\frac{2 \cdot I_{op}}{I_H} - 1 \right)}{\pi} \right] \quad (2)$$

In order to stabilize both the voltage and current at the output of the FC and improve the usefulness of the DC-DC converter, an L-C filter was connected between the FC and the converter. The filter consists of an inductor, L_x , in series with the fuel cell and a capacitor (C_x) in parallel with the converter input. The inductor stabilizes the current while the capacitor

does the same with the voltage. The equations describing the filter are given by (3) and (4).

$$\frac{dI_{FC}}{dt} = \frac{1}{L_x} (V_{FC} - V_{CX}) \quad (3)$$

$$\frac{dV_{CX}}{dt} = \frac{1}{C_x} (I_{FC} - I_L \cdot \delta) \quad (4)$$

The variable V_{FC} is the fuel cell voltage, which is a function of the fuel cell current, I_{FC} . V_{CX} is the capacitor voltage and I_L is the current that goes through the inductance of the converter. δ represents the time-varying behavior of the MOSFET used in the converter, i.e., $\delta = 1$ when the transistor is conducting (see Fig. 4(a)) and $\delta = 0$ when it is not conducting (see Fig. 4(b)).

B. Buck-Boost Converter and State-Space Model

The buck-boost converter transforms a D.C. voltage from one level to another. Specifically, the buck-boost can step up (boost) or step down (buck) the input voltage. The circuit used consists of a transistor used as a switch, an inductance, a capacitance, and a diode as shown on Fig. 3. The key element to the operation of this converter is the duty cycle D of the transistor. The duty cycle is defined as the ratio of the time the transistor is in the “on” state to the switching period. In our case this can also be stated as the average value of δ over one switching period. By controlling the duty cycle we can control the output voltage of the converter.

Equivalent configurations for the system during the “on” and “off” intervals of the buck-boost converter are shown in Fig. 4(a) and 4(b). The equivalent system during each one of these intervals was analyzed in order to obtain expressions dependent on the value of δ .

The equations describing the behavior of the converter while the transistor is “on” are:

$$\frac{dI_L}{dt} = \frac{1}{L} V_{cx} \quad (5)$$

$$\frac{dV_c}{dt} = -\frac{I_a}{C} \quad (6)$$

The equations describing the behavior of the converter while the transistor is “off” are:

$$\frac{dI_L}{dt} = -\frac{1}{L} V_c \quad (7)$$

$$\frac{dV_c}{dt} = \frac{1}{C} (I_L - I_a) \quad (8)$$

V_c is the output voltage of the converter, which is connected as input to the armature of the motor, I_a is the armature current, which is determined by the motor and L and C are the converter’s capacitance and inductance. In matrix form,

$$\mathbf{x} = [I_L \quad V_c]^T, \quad \mathbf{u} = [V_{cx} \quad I_a]^T, \quad \mathbf{y} = V_c \quad (9)$$

We can represent the converter while the transistor is “on” as

$$\dot{\mathbf{x}}_{ON} = \mathbf{0}_{2 \times 2} \mathbf{x} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{-1}{C} \end{bmatrix} \mathbf{u}, \quad \mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \mathbf{x} \quad (10)$$

While it is “off” it can be represented as

$$\dot{\mathbf{x}}_{OFF} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{C} \end{bmatrix} \mathbf{u}, \quad \mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \mathbf{x} \quad (11)$$

Since the converter is switching over time between the “on” and “off” state according to the value of δ , we can write

$$\begin{aligned} \dot{\mathbf{x}} &= \delta \dot{\mathbf{x}}_{ON} + (1 - \delta) \dot{\mathbf{x}}_{OFF} \\ &= \begin{bmatrix} 0 & \frac{(\delta-1)}{L} \\ \frac{(1-\delta)}{C} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{\delta}{L} & 0 \\ 0 & \frac{-1}{C} \end{bmatrix} \mathbf{u}, \quad \mathbf{y} = V_c = [0 \quad 1] \mathbf{x} \quad (12) \end{aligned}$$

C. DC Permanent Magnet Motor State-Space Model

The equations that describe the behavior of the DC motor are

$$\frac{dI_a}{dt} = \frac{1}{L_a} [V_a - R_a I_a - k_e \omega_m] \quad (13)$$

$$\frac{d\omega_m}{dt} = \frac{1}{J} [k_e I_a - B_m \omega_m - T_L] \quad (14)$$

In matrix form,

$$\mathbf{x} = [I_a \quad \omega_m]^T, \quad \mathbf{u} = [V_c \quad T_L]^T, \quad \mathbf{y} = \omega_m \quad (15)$$

$$\dot{\mathbf{x}} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{-k_e}{L_a} \\ \frac{k_e}{J} & \frac{-B_m}{J} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & \frac{-1}{J} \end{bmatrix} \mathbf{u}, \quad \mathbf{y} = [0 \quad 1] \mathbf{x} \quad (16)$$

III. DUTY CYCLE CONTROL

A. Duty Cycle Range

The range of existence of the duty cycle varies with different converter topologies. The following derivation, based on steady state analysis, shows the range of existence of the duty cycle for a buck-boost converter. The derivation is valid only for a fuel cell with minimum values of voltage and current equal to zero. Z is defined as the internal impedance of the fuel cell, which in steady state is the same as the input impedance of the converter (17). R is the impedance of the load (18):

$$Z = \frac{V_i}{I_i} \quad (17), \quad R = \frac{V_o}{I_o} \quad (18)$$

The input and output relationship of the converter is defined as a function of the duty cycle:

$$V_o = g(D) \cdot V_i \quad (19), \quad I_i = g(D) \cdot I_o \quad (20)$$

Using equations (17), (18), (19) and (20) we obtain:

$$[g(D)]^2 = \frac{R}{Z} = \left[\frac{-D}{1-D} \right]^2 \quad (21)$$

Solving for D we obtain:

$$D = \frac{1}{1 \mp \sqrt{\frac{Z}{R}}} \quad (22)$$

There are two solutions to (22). The negative root of the equation gives negative duty cycle for values of Z higher than R , which is physically meaningless. The duty cycle does not exist for every possible value of Z and R . On the other hand, the positive root of (22) gives a duty cycle between zero and one for every value of Z and R . Therefore the practical solution for D is:

$$0 \leq D = \frac{1}{1 + \sqrt{\frac{Z}{R}}} \leq 1 \quad (23)$$

For a DC motor, the impedance seen by the converter is:

$$R = R_a + \frac{K_e^2 \omega_m}{T_L + B_m \omega_m} \quad (24)$$

B. Control Implementation

This paper proposes a control system based on the control of the duty cycle of the DC to DC converter’s transistor. A pulse width modulator (PWM) is used to relate the motor’s angular speed to the converter’s duty cycle. A sawtooth signal with a frequency of 1kHz was used as the modulating signal. The modulated output of the compensator determines the duty cycle needed for the system to reach the established setpoint.

A proportional integral compensator (PI) was used. While the proportional part amplifies the signal, the integral part reduces the error in steady state. The signal that enters the compensator is the error – the difference between the setpoint and the motor’s current angular speed.

If the system were to be implemented, the actual shaft speed would be measured by a tachogenerator connected to the shaft of the motor. This device develops an output voltage that is linearly proportional to its angular speed input. This voltage is then appropriately amplified in order to be compared with the reference signal. The resulting difference between these two voltages is passed through a gain before it is fed to the PWM control loop.

IV. OBSERVER DESIGN

A benefit of having modeled the converter and the motor in state-space is the possibility of using linear state-space theory to design observers for these subsystems [17]. The motor constitutes a Linear Time-Invariant (LTI) system, for which the design of a linear state observer is straightforward. On the other hand, the switching action of the transistor on the converter makes it a non-LTI system, but still the methodology described in Chapter 8 of [17] produces acceptable approximate results that can be improved further by simple means.

Equation (25) is called the Bass-Gura formula for the observer gain vector.

$$L = \left(\begin{bmatrix} a_1 & a_2 & \dots & a_{n-1} & 1 \\ a_2 & a_3 & \dots & 1 & 0 \\ \vdots & 1 & \ddots & \vdots & \vdots \\ a_{n-1} & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{bmatrix} Q_{(A,C)} \right)^{-1} \begin{bmatrix} \alpha_0 - a_0 \\ \alpha_1 - a_1 \\ \alpha_2 - a_2 \\ \vdots \\ \alpha_{n-1} - a_{n-1} \end{bmatrix} \quad (25)$$

In (25), $Q_{(A,C)}$ is the observability matrix. For an observable system (i.e., with nonsingular $Q_{(A,C)}$), this formula provides us with a gain vector that allows us to place the eigenvalues of an observer at the locations determined by (26), which is known as the desired characteristic polynomial.

$$s^n + \alpha_{n-1}s^{n-1} + \dots + \alpha_1s + \alpha_0 \quad (26)$$

We will obtain the coefficients $\{a_0, a_1, \dots, a_n\}$ from the denominator of the transfer function $\frac{Y}{U} = C(sI - A)^{-1}B$ for both the converter and the motor. For the motor, these values are found to be

$$a_0 = (k_e^2 + B_m R_a) / J L_a \quad (27)$$

and

$$a_1 = (B_m L_a + J R_a) / J L_a \quad (28)$$

which yields the observer gain vector

$$L_M = \begin{bmatrix} \frac{J}{k_e} \left(\alpha_0 - \frac{k_e^2 + B_m R_a}{J L_a} \right) - \frac{J R_a}{k_e L_a} \left(\alpha_1 - \frac{J R_a + B_m L_a}{J L_a} \right) \\ \alpha_1 - \frac{J R_a + B_m L_a}{J L_a} \end{bmatrix} \quad (29)$$

Similarly, for the voltage converter, we find that

$$a_0 = (\delta - 1)^2 / LC \quad (30)$$

and

$$a_1 = 0$$

After substituting these values into the Bass-Gura formula, we obtain the following expression:

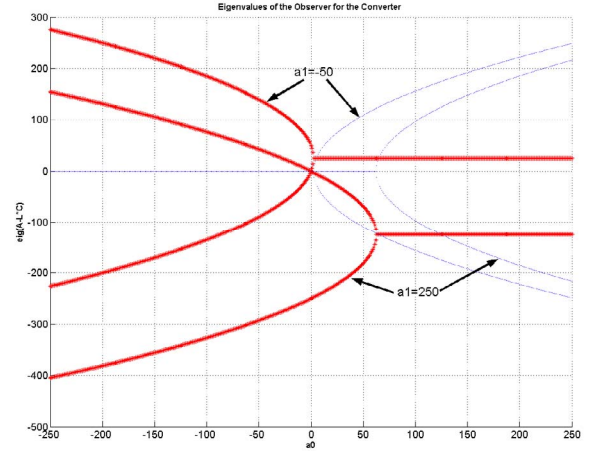


Fig.5. Plot of observer eigenvalues vs. desired characteristic polynomial coefficients

$$L_C = \begin{bmatrix} \frac{C}{1 - \delta} \left(\alpha_0 - \frac{(\delta - 1)^2}{LC} \right) \\ \alpha_1 \end{bmatrix} \quad (31)$$

It was found that the eigenvalues of the observer for the DC converter did not match closely the eigenvalues specified on its desired characteristic polynomial. In order to investigate into this and correct for the undesired effects, the observer's eigenvalues were plotted against the values of α_0 while keeping α_1 fixed. The results are shown on Fig. 5. The thick lines show the real part of the eigenvalues, the thin lines show the imaginary part. Two plots are shown, with α_1 fixed at two different values in order to show its effect on the eigenvalues. We can see that a value of α_1 large enough must be selected such that by adjusting α_0 we can have negative, real eigenvalues. Fig.4 helps us in determining such values for α_0 and α_1 . In our example shown in fig. 5, only the value of $\alpha_1 = 250$ gives us negative eigenvalues. The values of real, negative eigenvalues exist only between $\alpha_0 = 0$ and $\alpha_0 = 60$.

V. THEORETICAL CALCULATIONS AND RESULTS

We simulated the proposed system using the Simulink software from Matlab. A commercial FC from the Industry Horizon Fuel Cell Technologies was used. The terminal characteristics of the chosen FC are $V_h = 22V$, $I_h = 8A$, and $P_{max} = 100W$. The L-C filter values are: $L_x = 0.1 H$ and $C_x = 400 \mu F$. The Buck-Boost component values are $C = 400 \mu F$ and $L = 0.1 H$. The D.C. motor specifications and constants are: $120V$, $9.2A$, $1500rpm$, $R_a = 1.5 \Omega$, $L_a = 0.02 H$, $K_e = 0.621V/(rad/sec)$, $J = 0.02365Kg \cdot m^2$, and $B_m = 0.002387 Nm/(rad/sec)$. The controller gains are K_p and K_i for proportional gain and integral gain, respectively. These gains were selected based on trial and error.

Motor driving a load whose torque is $TL = KL \cdot \omega m$. The torque constant is $KL = 0.0135 \text{ Nm/(rad/sec)}$. The system response is shown on fig. 6. The input of the system is a series of exponential steps added at different intervals. These steps simulate a soft change in speed, like a car driver pushing the pedal smoothly. When the motor starts with initial conditions equal to zero, there is a small initial burst in speed, after which the rotor reaches the desired speed setpoint. After running at a certain constant speed, the reference is changed and the motor quickly follows the change with no overshoot. The motor cannot spin in the opposite direction because the converter voltage output has one polarity and no voltage inverter was implemented. The small delays between reference and response could be shortened or avoided if another control technique would be implemented. For example, using the observers one can implement a state feedback controller with possibly better performance.

Motor driving a load whose torque is $TL = KL \cdot \omega m^2$. The torque constant $KL = 0.0135 \text{ Nm/(rad/sec)}$. The system response is shown on fig. 7. The input to the system is also a series of exponential steps. The best controller in this case was the PI, with $Kp=500$ and $Ki=1500$. When the motor starts from zero speed there is a large overshoot in velocity of about 100%. All the setpoints are reached in less than 5 seconds.

Observer performance

Figures 8 through 11 show plots of the observer state trajectory of different states and their estimate. The plots were generated with observer initial conditions different than zero so that some error is seen and the convergence speed could be estimated. The observer reaches the state trajectory in less than 0.15 seconds. After this initial transient, the estimated states and the real states are identical, even when the setpoint is changed.

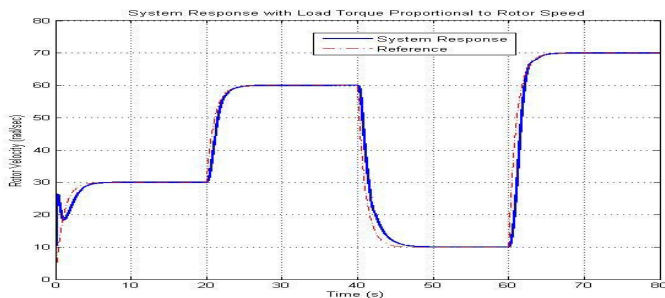


Fig. 6. DC motor speed vs. time, with torque proportional to speed.

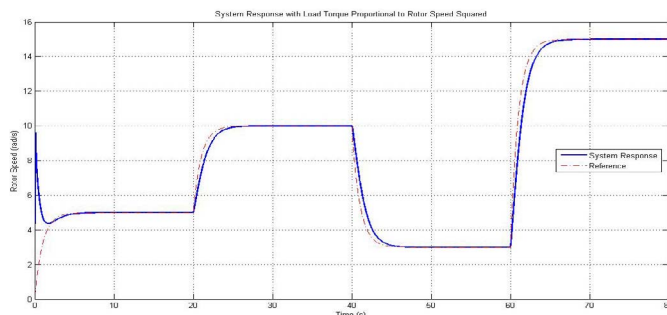


Fig. 7. Motor speed vs. time, with torque relative to the square of the speed.

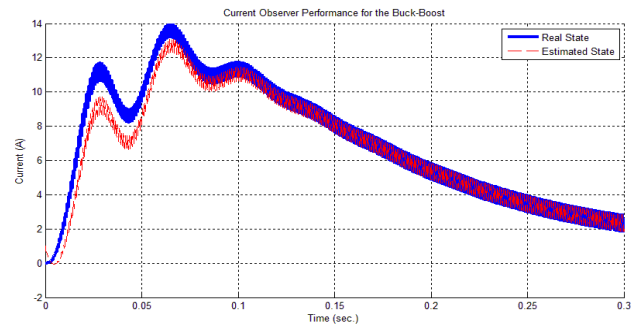


Fig. 8. Observer performance 1: Buck-Boost inductor current

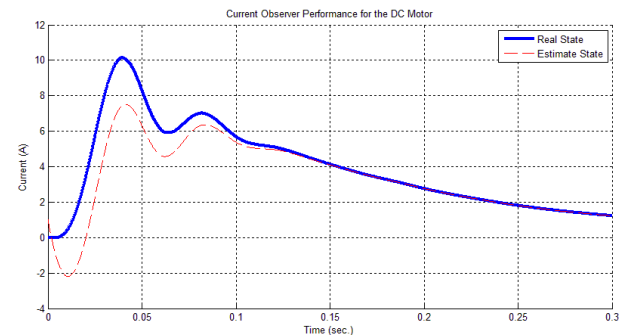


Fig. 9. Observer performance 2: DC motor armature current

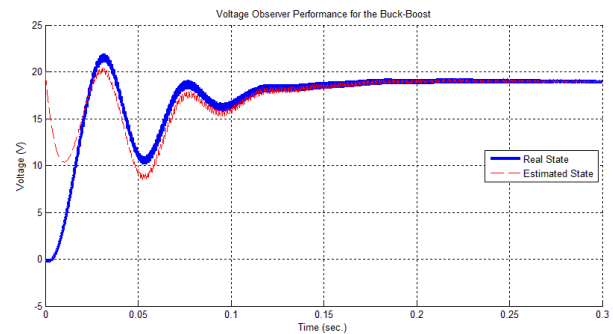


Fig. 10. Observer performance 3: Buck-Boost capacitor voltage

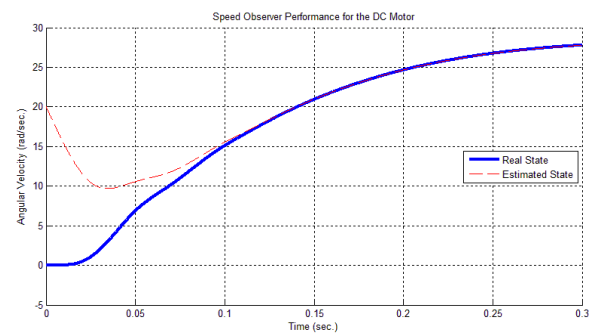


Fig. 11. Observer performance 4: DC motor shaft speed

VI. CONCLUSION

A system that controls a DC motor speed using a fuel cell was presented and validated. The state observers for the buck boost and the DC motor were presented and validated. Although some complications arose with the PI controller, like the overshoot, the next step is to implement state feedback to control the DC motor.

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